

Invasion of cooperation in scale-free networks: Accumulated vs. average payoffs

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Abstract

It is well known that cooperation cannot be an evolutionary stable strategy for a non-iterative game in a well-mixed population. In contrast, structured populations favor cooperation since cooperators can benefit each other by forming local clusters. Previous studies have shown that scale-free networks strongly promote cooperation. However, little is known about the invasion mechanism of cooperation in scale-free networks. To study microscopic and macroscopic behaviors of cooperators' invasion, we conducted computational experiments of the evolution of cooperation in scale-free networks where, starting from all defectors, cooperators can spontaneously emerge by mutation. Since the evolutionary dynamics are influenced by the definition of fitness, we tested two commonly adopted fitness functions: accumulated payoff and average payoff. Simulation results show that cooperation is strongly enhanced with the accumulated payoff fitness compared to the average payoff fitness. However, the difference between the two functions decreases as the average degree increases. Moreover, with the average payoff fitness, low-degree nodes play a more important role in spreading cooperative strategies compared to the case of the accumulated payoff fitness.

Author Summary

Social relationships among human individuals can be represented as a network. It is known that social networks are not randomly structured, but have a heavy tailed connectivity distribution where only a few nodes collect a large amount of connections. It has been reported that this social network structure helps cooperative behavior spread in society. However, previous studies typically assumed that society was initially occupied half by cooperators and half by non-cooperators. This assumption cannot help explain how cooperation can start in the first place. This study shows, via computer simulations, that cooperation can spontaneously arise and spread in the social network. We tested two commonly adopted fitness functions, accumulated payoff and average payoff. With the accumulated payoff fitness, highly connected nodes (called 'hubs') played an important role in spreading cooperative behavior. In contrast, with the average payoff fitness, low-degree nodes contributed more to the spread of cooperative behavior. These results imply that what makes society cooperative greatly depends on how individuals receive benefits from others.

Introduction

The emergence of cooperation is one of the challenging problems in both the biological and social sciences. Cooperators benefit others by incurring some costs to themselves while defectors do not pay any costs. Therefore, cooperation cannot be an evolutionary stable strategy for a non-iterative game in a well-mixed population. This relationship between cooperators and defectors is well parameterized in the Prisoner's Dilemma game (PD) [1]. In PD, two individuals decide whether to cooperate or defect simultaneously. They both obtain R for mutual cooperation or P for mutual defection as payoffs. If one selects cooperation

and the other selects defection, the former receives S for being the “sucker” of the defection, while the latter receives T as a reward, which is the “temptation” to defect. The order of the four payoffs is $T > R > P > S$ in PD.

Nowak and May were the first to reveal that spatial structures are required for the evolution of cooperation [2]. Recently, spatial structures have been mapped to suitable network topologies and the evolution of cooperation has been investigated through the analysis of PD played on those network topologies [3–8]. In this context, the effect of spatial structures required for the emergence of cooperation is referred to as network reciprocity, which has come to be recognized as one of the most important factors for the emergence of cooperation [9].

On such spatial structures, cooperators can form clusters and thereby reduce the risk of exploitation by defectors. In particular, it has been reported that scale-free networks strongly promote the evolution of cooperation [8]. If a cluster of cooperators occupies hub areas in a scale-free network, the payoffs for these cooperators are considerably higher than those for other individuals, and thus they can spread their cooperative strategy quickly to the entire network. In contrast, if a cluster of defectors occupies hub areas, this cluster of defectors is quite vulnerable and is easily replaced by cooperators. These effects explain why cooperation is likely to evolve on scale-free networks.

However, it is still unknown how cooperator-dominated hubs could arise in society that initially has few cooperators. It was assumed in [8] that the initial state of the social network was made of half cooperators and half defectors. This setting has since been adopted by other studies on evolutionary models of cooperation as described in [10, 11]. This assumption does not explain how one cooperator that emerged by mutation could increase in number and invade into the population initially filled by defectors. This invasion dynamics has already been investigated for square lattice network topologies [12, 13]. In contrast, little is known about such invasion dynamics on scale-free networks, although the fixation probability on scale-free networks has been briefly reported in [6].

Moreover, there is room for further investigation about how the evolution of cooperation depends on the accumulated payoff fitness, which is usually assumed in the previous studies mentioned above. The primary reason that cooperation is strongly promoted in scale-free networks is that cooperative hubs can gain extremely high payoffs compared to the other nodes because the payoffs are accumulated. In the meantime, if averaged payoffs are used [14–17] or some costs are incurred to maintain links [5], the evolution of cooperation is strongly inhibited. In this way, the evolutionary dynamics are greatly influenced by the definition of fitness.

In this paper, we performed computer simulations of the evolution of cooperation in scale-free networks where the initial population is all defectors but cooperators can spontaneously arise by mutation. In these simulations, we tested two commonly adopted fitness functions: accumulated payoff and average payoff. The purpose of this study is to reveal the microscopic and macroscopic behaviors of the cooperators’ invasion into the network, and how they differ between the two fitness assumptions.

Results/Discussion

Macroscopic dynamics of cooperator invasion

First, we focus on macroscopic dynamics of the cooperators’ invasion. We compared simulation results for the two fitness conditions (accumulated and average payoff fitnesses). For each fitness condition, we conducted simulations by varying two parameters: the temptation to defect (b , from 1.1 to 2.0) and the average node degree (\bar{k} , from 2 to 8). The other parameter sets used in the simulations were $N = 5,000$ (population size) and $m = 0.005$ (mutation probability). See the **Model** section for the algorithm of the simulation. Results are shown in Fig. 1, which clearly shows the effects of b and \bar{k} , as well as the difference between the two fitness conditions.

Basically, cooperation is greatly enhanced in the accumulated payoff fitness condition than in the

average payoff fitness condition, as previous studies have reported [14]. This is because cooperators occupying highly connected nodes can gain much greater payoffs than others, as long as the temptation to defect b is not too large. However, our results show that, if b is large (e.g., $b > 1.8$, $\bar{k} = 4$), cooperators may not be able to occupy hub nodes and therefore the cooperation is not promoted any more.

Moreover, in the accumulated payoff fitness condition, cooperation *decreases* as the average degree increases, which disagrees with what was originally reported in [8]. This disagreement is due to the difference in model settings: In the model used in [8], the initial population was filled by half cooperators and half defectors, so having a higher average degree made it easier for cooperators to form a cluster initially by themselves. In contrast, our model assumes that the initial condition is full of defectors, and that cooperators appear only by mutation. In this model setting, it is easier for cooperative clusters to form if the average degree is low, because the sparsity of the network helps lower the chance for an emerging cooperative cluster to be dominated by surrounding defectors.

In contrast, cooperation *increases* as the average degree also increases in the average payoff fitness condition. Therefore, the difference between the two functions decreases as the average degree increases. This is because the fitness difference between cooperators and defectors is weakened as the average degree increases. In that case, the chance for cooperators to survive increases. This may allow the existing cooperators to connect with each other and then form a group.

In brief, our results showed an interesting difference between the two fitness conditions, in terms of the effect of average node degrees. These findings rely on our unique model settings in which cooperators are initially non-existent and they spontaneously arise by mutation.

Microscopic dynamics of cooperators' invasion

Next, we investigate the microscopic dynamics of cooperator invasion. We used $\bar{k} = 4$ and $b = 1.2$ as a representative parameter setting throughout this section, because the general trend was consistent even if they are varied and also because these values were what other studies also used (e.g., [14]). Figure 2 shows histograms of strategy propagation events, plotted over the degrees of source and destination nodes. As seen in Fig. 2 C and D, only lower-degree nodes can change higher-degree nodes' strategies in the average payoff fitness condition. As a result, cooperation tends to spread more frequently from lower degree nodes in the average payoff fitness condition (Fig. 2C) than in the accumulated payoff fitness condition (Fig. 2A). This is because the benefit of being hubs for cooperators disappears in the average payoff fitness condition, as discussed above. In contrast, a relatively wide range of node degrees can cause a change in the strategy of neighbors although the strategy of hub nodes cannot be changed in the accumulated payoff fitness condition (Fig. 2 A and B).

To investigate the effects of the local surrounding environment for the propagation of cooperation, we also plotted histograms of strategy propagation events over the degree of the source node and its neighbors' state ratio (1 = fully cooperative neighborhood, 0 = fully defective neighborhood) in Fig. 3. In the accumulated payoff fitness condition, a node with any degree can change its neighbors' strategy in general. Moreover, as neighbors' state ratio becomes greater, the frequency of strategy change tends to be greater because such a high cooperation ratio contributes to raise the fitness of the source's node. In contrast, in the average payoff fitness condition, only low node degrees with any neighbors' ratio tend to be a cause of change in the strategy of neighbors as shown in Fig. 3C. In the case of high source node degrees in Fig. 3C, cooperators are easily invaded by defectors because defectors simply can get high payoff from such a high ratio of cooperators, resulting in no existence of defectors with a high neighbor ratio of cooperators as shown in Fig. 3D.

Case of donation game

In all the simulations above, we adopted a "weak" PD setting where the sucker's payoff (S) is equal to the punishment (P), which is a common assumption made in many earlier studies (e.g., [2, 8]). However,

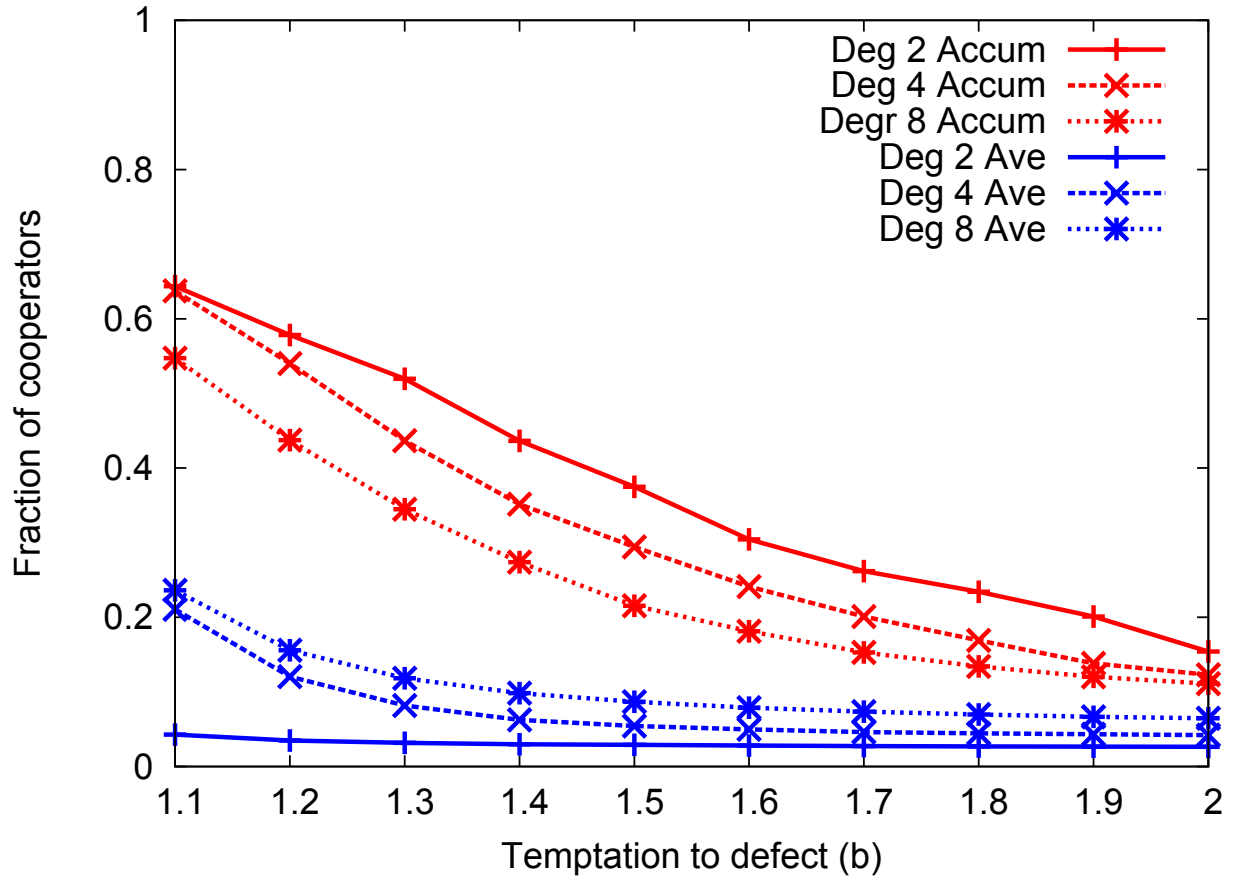


Figure 1. Fraction of cooperators against b in different average degree settings. The average of 10 independent simulation runs (the last 1,000 generations for each) is shown.

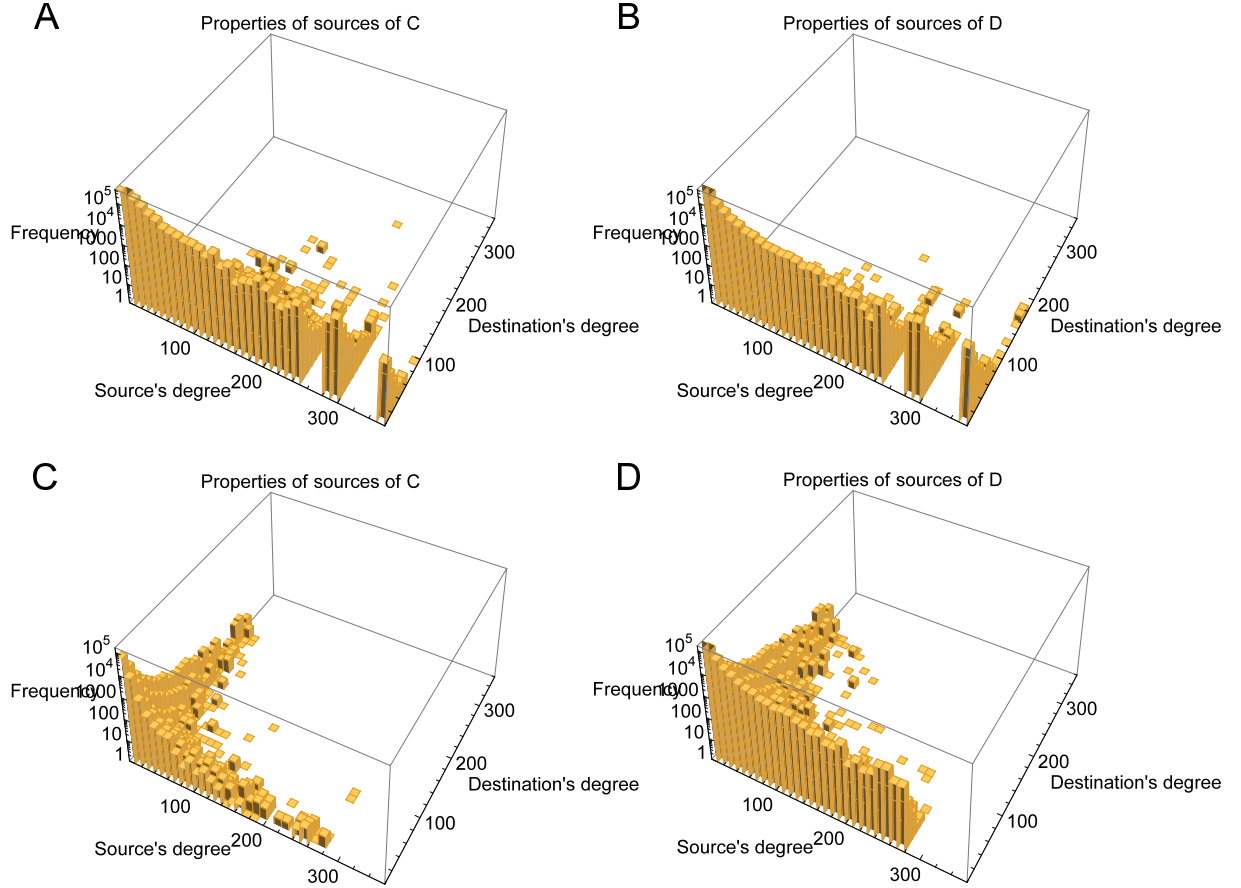


Figure 2. Frequency of strategy propagation events, plotted over the source node's degree and the destination node's degree in the accumulated payoff fitness condition (top: A and B) and the average payoff fitness condition (bottom: C and D). Cooperation propagation is on the left (A and C) and defection propagation is on the right (B and D). $N = 5,000$, $\bar{k} = 4$, $m = 0.005$, and $b = 1.2$. First 500 generations of 10 simulation runs are accumulated. Supplemental Fig. S1 shows that the first 500 generations are enough to see the invasion of cooperation.

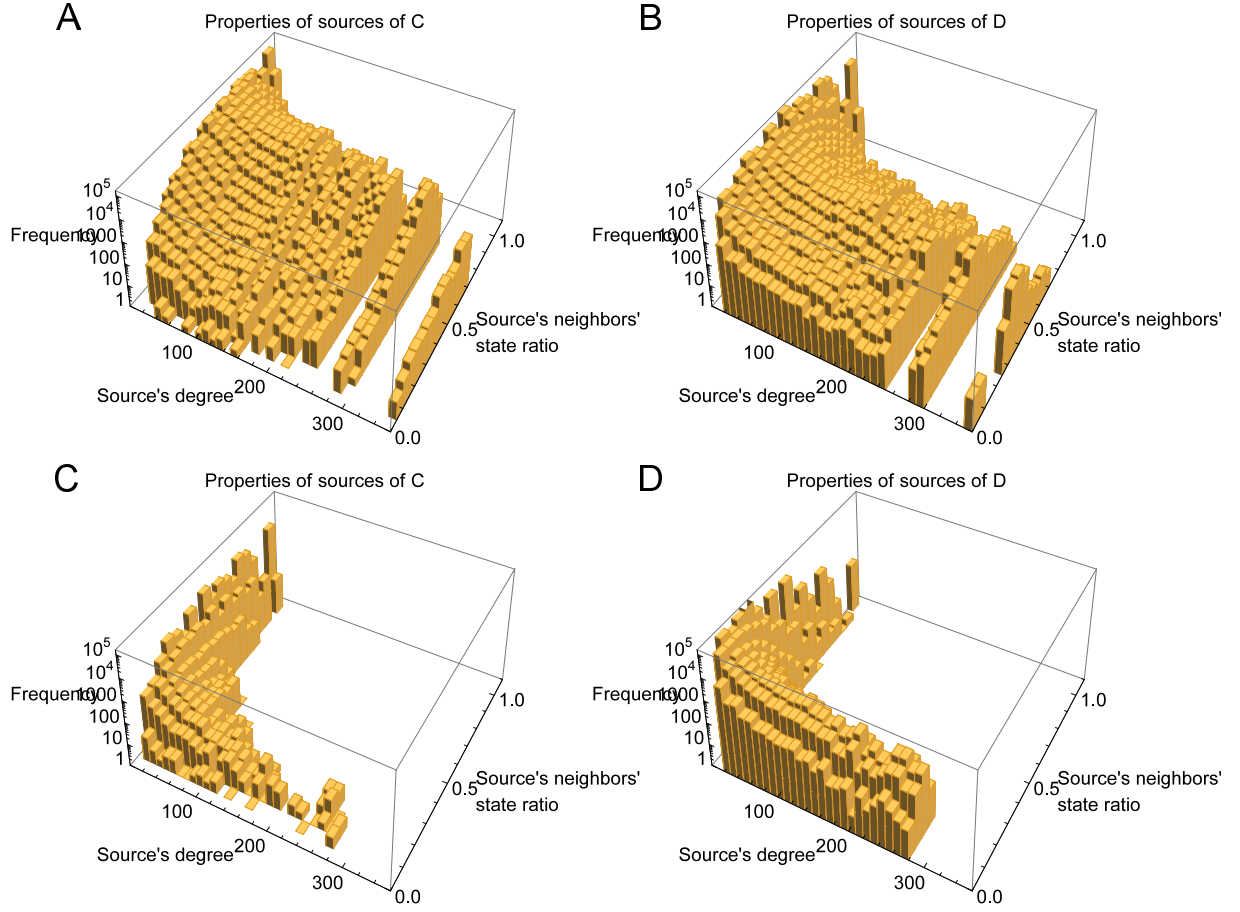


Figure 3. Frequency of strategy propagation events, plotted over the source node's degree and its neighbors' state ratio in the accumulated payoff fitness condition (top: A and B) and the average payoff fitness condition (bottom: C and D). Cooperation propagation is on the left (A and C) and defection propagation is on the right (B and D). $N = 5,000$, $\bar{k} = 4$, $m = 0.005$, and $b = 1.2$. First 500 generations of 10 simulation runs are accumulated. Supplemental Fig. S1 shows that the first 500 generations are enough to see the invasion of cooperation.

this assumption does not create an incentive for cooperators to switch their strategy to defection when they play a game with defectors.

In order to test the robustness of our findings in a “strong” PD setting with $T > R > P > S$ and $2R > T + S$, we conducted another set of simulations using the “donation game” model. The donation game is a special class of “strong” PD where each cooperator provides a benefit b to the other player by incurring cost c to himself, with $0 < c < b$. Thus, the payoff structure of the donation game is given by $T = b$, $R = b - c$, $P = 0$, and $S = -c$ [6]. For simplicity, we varied just one parameter b from 1 to 2, while letting $c = b - 1$. Figure 4 shows the fraction of cooperators in the simulations of the donation game. We find cooperation is greatly inhibited in the donation game compared to the weak PD. Cooperation sharply drops in both cases and the same tendency between average and accumulated payoff fitness conditions with the weak PD can only be seen in the low limited range of b ($b = 1.1$).

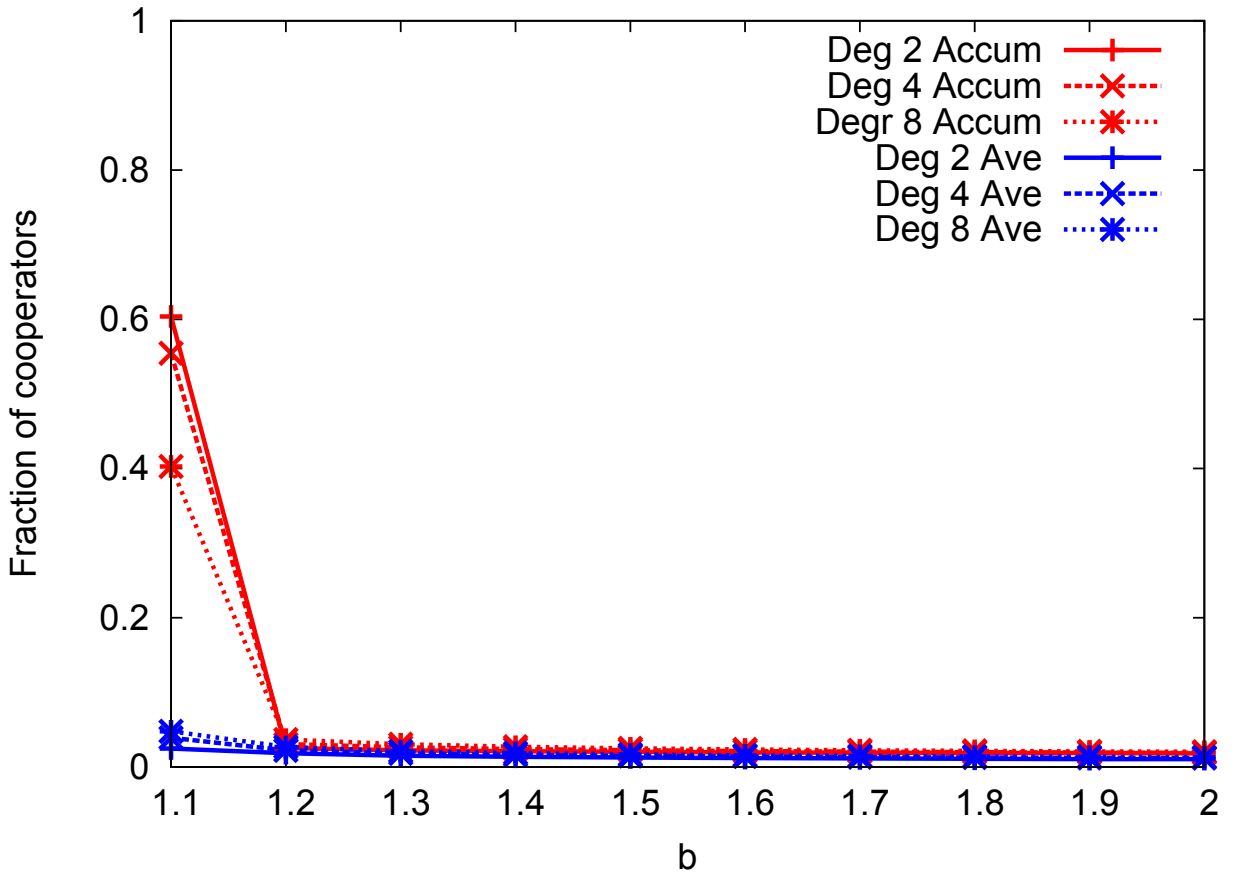


Figure 4. Fraction of cooperators in the case of the donation game. The average of 10 independent simulation runs (the last 1,000 generations for each) is shown.

In conclusion, it is known that scale-free networks strongly promote cooperation due to their heterogeneity. It is also known that this advantage is mostly lost when average payoffs are adopted. However, little is known about how cooperation can spread from initially rare cooperators in the two cases. Here we show that the fate of cooperation between the average and accumulated payoff fitness condition is very different depending on the average degree. In general, cooperation is promoted in the accumulated payoff fitness condition as previously found. However, the difference between accumulated and average

payoff decreases as the average degree increases, which is not commonly discussed in the previous studies. This implies that the evolution of cooperation on a network depends significantly on how game players are rewarded through their game play. Moreover, from the in-depth analysis of microscopic behaviors, we show that the relative importance of low degree nodes for the evolution of cooperation is much higher in the case of the average payoff, compared to the previously studied case of the accumulated payoff where hubs are known to play a major role in the propagation of cooperation.

Models

We consider the evolutionary dynamics of cooperators' invasion in scale-free networks. Barabási-Albert method is used for generating initial networks in simulations [18]. Then, each generated network is substantially randomized by the double-edge swap method while keeping the original degree distributions. Self loops and parallel edges are avoided during the randomization.

A network is made of N nodes occupied by individuals. Each node has its strategy classified as either C (cooperator) or D (defector). Each node i plays PD with all of its k_i neighbors. Initially, all individuals are defectors. Each node i plays the Prisoner's Dilemma game (PD) with all of its k_i neighbors. The payoffs of the game are calculated as follows. Both individuals obtain R for mutual cooperation and P for mutual defection. If one selects cooperation and the other selects defection, the cooperator obtains S as the sucker of defection, and the defector obtains T as the reward for temptation to defect. The order of the four payoffs is $T > R > P \geq S$ in typical PD. In the case that $P = S$, the game is called weak prisoner's dilemma. Following previous studies [2, 8], we set $P = 0$, $T = b$, $R = 1$, and $S = 0$, where $b > 1$ is the only control parameter. The payoff of individual i against its k_i neighbors is denoted by p_i . Here we consider two types of p_i : accumulated payoff and average payoff. The average payoff is obtained by dividing the accumulated payoff by k_i .

At the beginning of each simulation, one randomly selected individual x plays PD with its neighbors and obtains payoff p_x . Next, one randomly chosen neighbor of x , denoted by y , also plays PD with its neighbors and obtains payoff p_y . If $p_x < p_y$, individual x imitates individual y 's strategy with probability $(p_y - p_x)/[(T - S)k_{\max}]$ [8], where $k_{\max} = \max(k_x, k_y)$, for the accumulated payoff condition and $(p_y - p_x)/(T - S)$, for the average payoff condition. Finally, another randomly selected individual z (z might be the same as x or y) flips its strategy (C will become D and D will become C) by mutation with probability m . These operations consist of the one time step. We regard N time steps as one generation, in which all individuals are selected once, on average, for the strategy update and mutation.

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